An Inventory Recovery Model for an Economic Lot Sizing Problem with Disruption

Hawa Hishamuddin[*], R. A. Sarker, D. Essam
School of Engineering and Information Technology,
University of New South Wales,
ADFA Campus, Northcott Drive,
Canberra 2600, Australia
[*]**EMAIL: h.hishamuddin@student.adfa.edu.au,**

Abstract: Supply chains face risks from various unexpected events that make disruptions almost inevitable. This paper presents a disruption recovery model for a single stage production and inventory system, where finished product supply is randomly disrupted for periods of random duration. A production facility that manufactures a single product following the Economic Production Quantity policy is considered. The model is solved using a search algorithm combined with a penalty function method to find the best recovery plan. It is shown that the optimal recovery schedule is dependent on the extent of the disruption, as well as the back order cost and lost sales cost parameters. The proposed model is seen to be a very useful tool for manufacturers to make quick decisions on the optimal recovery plan after the occurrence of a disruption.

Keywords: Supply chain, Disruption, Inventory-production system, Economic lot size

I. Introduction

The supply chain is a system that consists of facilities or entities that are involved in transferring goods from supplier to customer. The activities in a supply chain have the role of transforming raw materials into finished products that are delivered to the end customer. Conventional supply chains are often designed to operate smoothly in a problem-free environment. However, in the real world, unexpected events such as machine breakdowns, transportation failures, labor strikes, and natural disasters are bound to happen and are often inevitable. This may cause disruptions at different levels of the supply chain, from the upstream to the downstream stages. Without a proper response to these events, a manufacturer would have to incur high additional costs to recover from the negative impacts of disruption. For instance, the 1995 earthquake that hit Kobe left vast damage to all transportation links in Kobe and nearly destroyed the world's sixth-largest shipping port. The 7.2 scale Richter quake terribly affected Toyota, where an estimated production of 20,000 cars, equivalent to $200 million worth of revenue was lost due to parts shortages [8].

Realizing the potential losses from such events, enterprises have recently shown a growing interest to incorporate risk management into their operations. Two tactics to deal with the risk of disruption include mitigation and contingency tactics [11]. A commonly practiced strategy for protecting against disruption is to hold additional inventory in the system for the entire period. Various related studies have been conducted for inventory models under the continuous review framework [5] [6] [12] and the periodic review framework [1] [11] [3]. These studies mainly design their inventory models to incorporate supply uncertainty occurrences by modifying the original non-disruption models. However, the majority result in stationary higher ordering quantities or bigger stock levels that would incur unnecessarily high holding costs for the long run. Thus, this motivates us to focus on disruption *recovery* strategies in developing our model.

Studies on optimal recovery strategies for disruptions are rather scarce. In the production and inventory literature with regards to the Economic Lot Scheduling Problem (ELSP), [4] and [10] proposed methods on how to recover from a schedule disruption. Xia *et al.* [13] developed a recovery strategy for an EPQ system subject to disruption in the form of parameter changes. The main aim was to minimize the disruption costs by incorporating penalty costs for deviations in the objective function. The original plan is recovered within short time windows spanning two to three production cycles. The work presented here is very much related to this paper.

In this paper, a recovery model for a single stage inventory system subject to disruption is presented. We consider a production facility that manufactures a single product in batches at a constant time interval following the Economic Production Quantity (EPQ) model. However, it is assumed that a random disruption occurs during a cycle, thus disabling the production to run as scheduled. After the disruption occurs, a specified duration, known as the recovery time window, is allocated to the production system to allow time to recover from the disruption. During the recovery duration, changes are made to the original production schedule to try to satisfy customer orders, where shortages may become a mix of backorders and lost sales. Similar to other disruption management models, the original production schedule is restored at the end of the recovery time window.

The production facility faces four types of costs: a setup cost, inventory holding cost, backorder cost, and lost sales cost. The objective of the model is to determine the optimal production
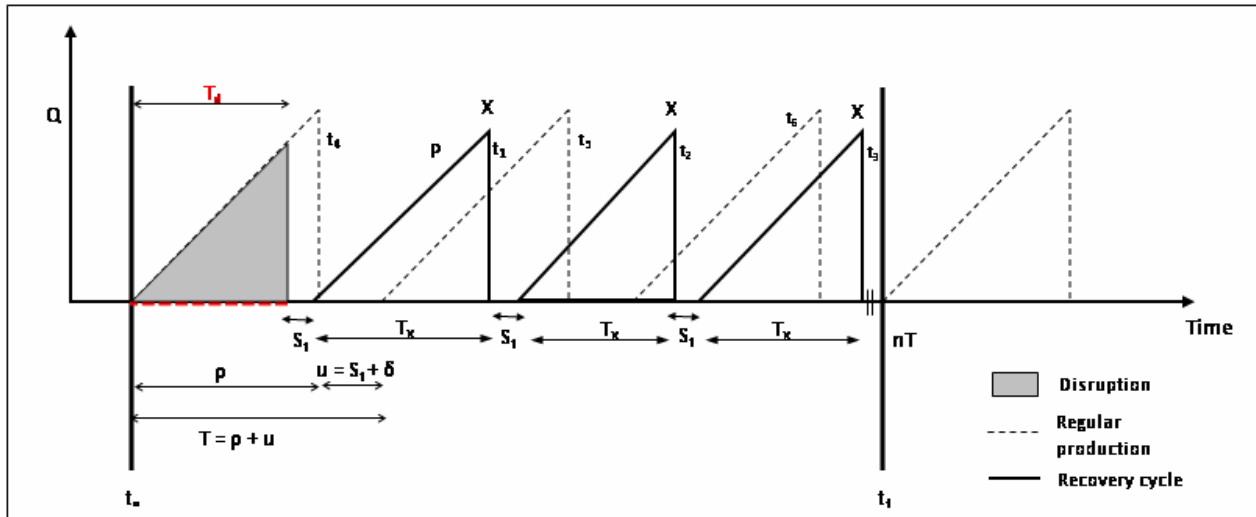
Figure. 1 Production Inventory Curve with Disruption

quantity for the cycles in the recovery time window so that the expected total cost is minimized. The results show that the optimal length of the recovery duration is dependent on the length of the disruption, as well as the relationship between the backorder and lost sales cost.

The remainder of this paper is structured as follows. The model formulation is proposed in Section 2. Section 3 suggests a possible method to solve the model and in Section 4, the related computational results are provided in the form of several numerical examples and an analysis of the model. Finally, Section 5 summarizes the paper and provides directions for future research.

II. Model Formulation

We assume that the current production-inventory system is run based on the well known Economic Production Quantity (EPQ) policy. Specifically, we consider the EPQ system similar to Sarker and Khan [7]. The system has a lot-for-lot delivery policy, where the optimal production lot size, (Q) is:

$$Q = \sqrt{\frac{2AP}{H}}$$

(1)

The notations (A, P, H) are defined in the next column. In the next sub-sections, a similar model accounting for disruption will be developed.

Disruption Recovery Model Formulation
In this section, a general cost model is developed for a production facility that experiences disruption, as explained in section 1. The disruption recovery time window concept considered in this paper was adopted from the works of [13]. However, our model assumes that the pre-disruption period is zero and the disruption randomly occurs such that it is not

known in advance. For simplicity, we set the recovery time window to be equal to n cycle times from the start of a disruption. During the recovery period, the production schedule is modified such that the length of n cycles in the recovery schedule is equal to n cycles in the original schedule. The only difference is that the recovery schedule includes the disruption length, T_d. In our model, n is made as a decision variable. Like other disruption management models, the term recovery is defined as restoring the original production schedule within a considerably short time period, while minimizing overall costs. The model is capable of determining the optimal manufacturing batch size for the production run in the recovery time window, so as to minimize the total cost for recovery, while trying to fulfill customer demands and other system constraints. The problem is illustrated in Figure I. The notations used in developing the cost function are as follows:

A: setup cost for a cycle ($/setup)
D: demand rate for a product (units/year)
H: annual inventory holding cost ($/unit/year)
P: production rate (units/year)
Q: production lot size in the original schedule (units),
T_d: disruption period
u: production down time for a normal cycle (setup time + idle time), ($T - Q/P$)
t_e: start of recovery time window
t_f: end of recovery time window
T: production cycle time for a normal cycle (Q/D)
ρ: production up time for a normal cycle (Q/P)
B: unit back order cost per unit time ($/unit/time)
L: unit lost sales cost ($/unit)
X_i: production quantity for cycle i in the recovery window (units)
T_i: production up time for cycle i in the recovery window (X_i/P)

St: setup time for a cycle

δ: idle time for a cycle

When a disruption occurs, this will create a production delay in the system. This delay is dependent on the total disruption duration, T_d. Production can only resume after the disruption ends when the problem is rectified. Unsatisfied customer demand during this stock out period will be partially backlogged, where some will become backorders, which will be produced during the recovery time window, and the remainder will become lost sales. The backorder costs, *B*, will be a function of time delayed with units ($/unit/unit time). Additionally, lost sales may occur during any of the cycles in the recovery time window. One of the advantages of our model is the ability to decide on the amount of backorders and lost sales in each recovery cycle that provides the most cost effective solution.

Mathematical Representation

Let T_d be the disruption period occurring at the beginning of a cycle (see Figure 2). It is assumed that T_d is less than the normal production cycle time, *T*, for this model. After a disruption of T_d occurs, recovery takes place by utilizing the production idle times, *δ,* in the original schedule. The recovery time window will be *n* normal production cycle times from the start of disruption. We define the decision variable X_i as the production quantity for cycle *i* in the recovery time window and T_i as its respective production time, where *i*= 1, 2, …, *n*. However, in this paper we assume:

$$X_1 = X_2 = \dots = X, \text{ thus } T_1 = T_2 = \dots = T_x$$

The setup cost equation is rather straight forward and can be obtained by:

$= A \cdot (\textit{Number of setups})$

$= A \cdot n$

(2)

The inventory holding cost is equal to the unit inventory holding cost*, H,* multiplied by the total inventory during the recovery time, which is the area under the curve. This is calculated as:

$= H\left[\tfrac{1}{2}\, XT_x + \tfrac{1}{2}\, XT_x + \dots\right]$

$= \dfrac{H}{2} \cdot n \cdot X \cdot T_x$

$= \dfrac{H}{2} \cdot n \cdot X \cdot \dfrac{X}{P}$

$= \dfrac{H}{2P} \cdot n \cdot X^2$

(3)

Next, the backorder cost formulation can be derived by multiplying the unit backorder costs, *B,* with the backorder

units of each cycle *i* and it's time delay, given that the delay is a *positive* value:

$B\,[X \cdot (Delay_1) + X \cdot (Delay_2) + X \cdot (Delay_3) + \dots + X \cdot (Delay_n)]$

$= B\,[X \cdot \sum\limits_{i=1}^{n} Delay_i\,]$

(4)

The delay for backorders in each cycle is calculated below:

$Delay\ i = T_d + i \cdot S_t + i \cdot \left(\dfrac{X}{P}\right) - i \cdot \left(\dfrac{Q}{P}\right) - u(i-1)$

(5)

where all delays are non-negative.

Thus the backorder cost is:

$BX\left[\sum\limits_{i=1}^{n}\left(T_d + i \cdot S_t + i \cdot \left(\dfrac{X}{P}\right) - i \cdot \left(\dfrac{Q}{P}\right) - u(i-1)\right)\right]$

(6)

Finally, the lost sales cost is obtained as:

$= L\,(nQ - X - X - X - \dots)$

$= L\,(nQ - nX)$

$= Ln\,(Q-X)$

(7)

The sum of all the cost components above gives the total relevant costs of the recovery plan. The total cost function can be derived as below:

$TC(X,n) = (A \cdot n) + \left(\dfrac{H}{2P} \cdot n \cdot X^2\right) +$

$\left(BX\left[\sum\limits_{i=1}^{n}\left(T_d + i \cdot S_t + i \cdot \left(\dfrac{X}{P}\right) - i \cdot \left(\dfrac{Q}{P}\right) - u(i-1)\right)\right]\right)$

$+ (Ln(Q - X))$

Using the sum of powers rule, the above equation can be expanded to the following:

$TC(X,n) = (A \cdot n) + \left(\dfrac{H}{2P} \cdot n \cdot X^2\right) +$

$BX\left(\dfrac{n}{2}(n+1) \cdot S_t + \dfrac{n}{2}(n+1) \cdot \dfrac{X}{P} - \dfrac{n}{2}(n+1) \cdot \dfrac{Q}{P} - \dfrac{n}{2}(n+1) \cdot u\right)$

$+ BXn(T_d + u) + (Ln(Q - X))$

Therefore, the model (*P1*) is formulated as follows:

$MinTC\,(X,n) = (A \cdot n) + \left(\dfrac{H}{2P} \cdot n \cdot X^2\right) +$

$BX\left(\dfrac{n}{2}(n+1) \cdot S_t + \dfrac{n}{2}(n+1) \cdot \dfrac{X}{P} - \dfrac{n}{2}(n+1) \cdot \dfrac{Q}{P} - \dfrac{n}{2}(n+1) \cdot u\right)$

$+ BXn(T_d + u) + (Ln(Q - X))$

(8)

Subject to:

$X \leq Q$
(9)

$nX \leq P\,(nT - nS_t - T_d)$
(10)

$nX \geq nTD - n(Q - X)$
(11)

The objective function (8) comprises of the four cost components mentioned earlier, each separated in parenthesis. Constraint (9) requires that the production quantities in the recovery time window be less than the production quantity under the original schedule due to the delivery and transportation requirements. Constraint (10) represents the available production capacity and constraint (11) ensures that all the demand is accounted for.

By solving the above model for X_i subject to the constraints (9)-(11), one can obtain the optimal recovery plan for the production system under disruption. Without disruption, this model will reduce to the original EPQ model as presented earlier. The following theorems have been developed for the model:

Theorem 1. For a given A and H, if $B<<L$, the number of recovery cycles will be $\lceil n \rceil$, where $n = \dfrac{P \cdot T_d}{P(T - S_t) - Q}$.

Proof. From (10), $n \geq \dfrac{P \cdot T_d}{P(T - S_t) - X}$

(12)

From (9), $X \leq Q$

Substituting $X = Q$ in (12), we get

$$n \geq \frac{P \cdot T_d}{P(T - S_t) - Q}$$

(13)

So the value of $n = \dfrac{P \cdot T_d}{P(T - S_t) - Q}$

(14)

is the minimum number of cycles to fulfill the demand without any lost sales. As n must be an integer, we use $\lceil n \rceil$.

For $X < Q$, we can see that $P(T - S_t) - X \geq P(T - S_t) - Q$.

This means that $\dfrac{P \cdot T_d}{P(T - S_t) - X} < \dfrac{P \cdot T_d}{P(T - S_t) - Q}$, in which case,

there will be some lost sales. As we assume $B<<L$, the lost sales will incur higher costs. Therefore, it would be more optimal for $X = Q$. This proves that the number of recovery cycles will be $\lceil n \rceil$ for the case where $B<<L$.

Theorem 2. For a given A and H, if $B>>L$, the number of recovery cycles will be $n < \dfrac{P \cdot T_d}{P(T - S_t) - Q}$.

Proof. For $B>>L$, lost sales will be encouraged for quick recovery. That means $X < Q$, which indicates

that $\dfrac{P \cdot T_d}{P(T - S_t) - X} < \dfrac{P \cdot T_d}{P(T - S_t) - Q}$. This proves that when

$B>>L$, some demand will become lost sales, which results in

n to be less than $\dfrac{P \cdot T_d}{P(T - S_t) - Q}$

III. Solution Approach

In this model, the number of recovery cycles, n, has been set as an integer. It may be given as a user input or can be determined as part of the solution process. However, if one can fix the value of n, the solution process for *Model P1* will be easier. After rearranging (12), it can be shown that this constraint is independent of both n and X. Therefore it can be ignored from the optimization process. Based on the theorems presented earlier, our solution approach can be summarized as follows:

Step 0: Initialize the parameters
Step 1: Find n using (15) and set $n' = \lceil n \rceil$
Step 2: Solve *Model P1* for X using $n = n'$
Step 3: If $B<<L$, record the solution and go to Step 5
Step 4:   a) Set $K = 1$
        b) Set $n = n - K$
        c) Solve *Model P1*
        d) If $TC(K) < TC(K-1)$, set $K = K+1$, go to (b).
           Otherwise, record the solution and go to Step 5.
Step 5: Stop

The *Model P1* can be categorized as a non-linear constrained integer optimization problem and is solved using the penalty function method to find the optimal values of X.

The penalty method has been used widely in the literature for solving constrained optimization problems. The basic idea behind this method is to approximate a constrained optimization problem with a sequence of unconstrained problems that are easier to solve. This is achieved by adding a penalty in the objective function for infeasibility, which will increase the objective for any given constrained violation [9]. The technique used for solving our model is known as the *dynamic penalty function*, where the penalty parameter for   a

Table 1 Comparison of the results between the Penalty Method and LINGO for five test problems

	A	H	B	L	T_d	S_t	n	Q	X		TC		Error
									Penalty Method	LINGO	Penalty Method	LINGO	
Example 1	50	2.4	10	15	0.003	0	4	14433.76	14433.76	14433.76	1623.40	1623.40	0.00000
Example 2	50	2.4	10	15	0.003	0.000057	4	14433.76	14433.76	14433.76	1746.81	1746.81	0.00000
Example 3	20	1.2	10	15	0.003	0.000057	5	12909.94	12909.94	12909.94	1468.32	1468.32	0.00000
Example 4	20	1.2	100	1	0.003	0.000057	2	12909.94	8352.43	8352.43	11502.93	11502.93	0.00136
Example 5	50	2.4	100	1	0.003	0.000057	2	14433.76	10257.20	10257.20	11522.70	11522.70	0.00029

given violation increases as the search progresses. This property allows highly infeasible solutions at the beginning of the search, but eventually approaches to an optimal solution as the penalty parameter becomes larger. Since the points generated move to a final solution from outside the feasible region, this technique is also referred to as an *exterior penalty function method* [2]. A summary of the penalty function method proposed by Bazaraa and Shetty [2] is provided below.

Initialization Step: Let $\varepsilon > 0$ be a termination scalar. Choose an initial point X_1, a penalty parameter $\mu > 0$, and a scalar $\beta > 0$. Let $k = 1$ and proceed to the main step.

Main Step:
1. Starting with X_k, solve the following problem:
   Minimize $f(X) + \mu_k \alpha(X)$
   Subject to $\widetilde{x} \in X$
   Let X_{k+1} be an optimal solution, and go to step 2.
2. If $\mu_k \alpha(X_{k+1}) < \varepsilon$ stop; otherwise, let $\mu_{k+1} = \beta \mu_k$, replace k by $k+1$, and go to step 1.

Based on the proposed method above, the parameters to run the solution procedure were chosen with $\mu_1 = 0.1$ and $\beta = 10$. The starting point was taken as $X_1 = Q$. The penalty method procedure was coded in MATLAB and executed on an Intel Core Duo processor with 1.99 GB RAM and a 2.66 GHz CPU.

IV. Computational Experience

Some numerical examples are presented in this section to demonstrate the applicability of the model developed in this paper. Test problems were generated by arbitrarily changing the cost parameters (setup, holding, back order, and lost sales cost) as well as the disruption duration (see Table I). For a backorder cost that is significantly lower than the lost sales cost, it is found that all shortages will be backordered and the optimal X value is found to be equal to Q. However, when the backorder cost is significantly higher than lost sales costs, it is shown that there will be some amount of lost sales. In addition, the optimal production quantities in the recovery schedule, X, is found to be less than that of the original schedule, Q. The recovery duration, n, will be shorter for the second scenario compared to the first. A comparison of the

solutions was made by solving the same test problems using the LINGO 10.0 optimization software, where both X and n are variables. From the comparison results in Table I, it can be observed that the differences are negligible for all cases.

An analysis has been carried out to show the effect of increased backorder cost, B, on X, TC, backorder quantity and lost sales quantity when lost sales cost, L, is significantly low (see Figure II). With L fixed at \$1/unit and B increasing from \$10 to \$5000, it can be observed that the value of X decreases from Q to zero. In addition, the lost sales quantity is found to increase, while the backorder quantity decreases to zero. An explanation for this is that as the backorder cost becomes larger, it is more optimal to have lost sales rather than backorders in the recovery schedule. Thus, when a portion of the demand becomes lost sales, the quantity to be produced, X, becomes lower and the recovery duration, n, will become shorter.

Figure 2 Effect of B on X, backorder quantity, lost sales quantity and TC when L is fixed at \$1/unit
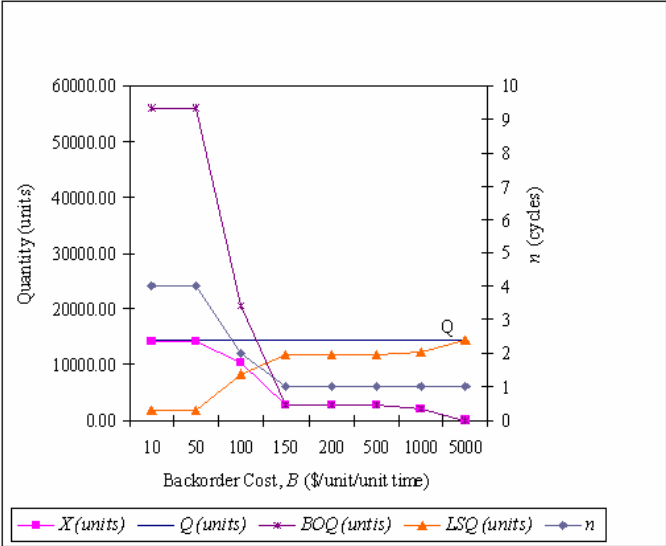
V. Conclusion

A recovery model for a single stage inventory system subject to disruption has been presented in this paper. The model determines the optimal production quantity and the number of cycles for recovery in order to minimize the total cost for recovery including setup costs, inventory holding costs and

shortage costs. The problem was formulated as a nonlinear constrained integer programming problem for which we chose the penalty function method as the solution technique. The results of several test problems were compared to that of a standard optimization software to examine the quality of the solution. Computational results were presented for different sets of examples and an analysis of the model was incorporated to provide better understanding of the model's applicability. From the analysis, it is shown that the optimal recovery schedule is dependent on the length of the disruption,

as well as the relationship between the backorder and lost sales cost. The proposed model is believed to be a very useful tool to help manufacturers make prompt and accurate decisions on the optimal recovery plan when a disruption occurs in the production system. For future research, work is under progress for a similar model that allows different values of X for each cycle in the recovery window. We will also consider the case where the disruption occurs in the middle of a production cycle, where a certain quantity of products has already been produced.

References

[1] Arreola-Risa, A. and DeCroix, G.A., 1998. Inventory Management under Random Supply Disruptions and Partial Backorders, *Naval Research Logistics,* Vol. 45, 687-703.

[2] Bazaraa, M.S. and Shetty, C.M., 1979. *Nonlinear Programming: Theory and Algorithms,* John Wiley & Sons, Atlanta, Georgia.

[3] Chopra, S., Reinhardt, G., Mohan, U., and 2007. The Importance of Decoupling Recurrent and Disruption Risks in a Supply Chain, *Naval Research Logistics,* Vol. 54, 5, 544-555.

[4] Gallego, G., 1994. When Is a Base Stock Policy Optimal in Recovering Disrupted Cyclic Schedules?, *Naval Research Logistics,* Vol. 41, 317-333.

[5] Parlar, M. and Berkin, D., 1991. Future Supply Uncertainty in Eoq Models, *Naval Research Logistics,* Vol. 38, 107-121.

[6] Parlar, M. and Perry, D., 1995. Analysis of a (Q, R, T) Inventory Policy with Deterministic and Random Yields When Future Supply Is Uncertain, *European Journal of Operational Research,* Vol. 84, 431-443.

[7] Sarker, R.A. and Khan, L.R., 2001. An Optimal Batch Size under a Preriodic Delivery Poilicy, *International Journal of Systems Science,* Vol. 32, 9, 1089-1099.

[8] Sheffi, Y., 2005. *The Resilient Enterprise: Overcoming Vulnerability for Competitive Advantage,* The MIT Press, Cambridge, Massachusetts.

[9] Smith, A.E. and Coit, D.W., 1995. Penalty Functions, in *Handbook of Evolutionary Computation.* Baeck, T., Fogel, D., and Michalwicz, Z., A joint publication of Oxford University Press and Institute of Physics Publishing, Pittsburgh, Pennsylvania.

[10] Tang, L.C. and Lee, L.H., 2005. A Simple Recovery Strategy for Economic Lot Scheduling Problem: A Two-Product Case, *International Journal of Production Economics,* Vol. 98, 97-107.

[11] Tomlin, B., 2006. On the Value of Mitigation and Contingency Strategies for Managing Supply Chain Disruption Risks, *Management Science,* Vol. 52, 5, 639-657.

[12] Weiss, H.J. and Rosenthal, E.C., 1992. Optimal Ordering Policies When Anticipating a Disruption in Supply or Demand, *European Journal of Operational Research,* Vol. 59, 2, 370-382.

[13] Xia, Y., Yang, M.-H., Golany, B., Gilbert, S.M., and Yu, G., 2004. Real-Time Disruption Management in a Two-Stage Production and Inventory System, *IIE Transactions,* Vol. 36, 111-125.

Back Ground of Authors

Hawa Hishamuddin is a doctoral candidate at the School of Engineering and Information Technology, University of New South Wales, ADFA Campus at Canberra, Australia. She received her Bachelor's degree in 2003 from the School of Industrial Engineering, Purdue University, USA. Her current research interests include optimal policies for supply chain systems with disruption.

Ruhul Sarker obtained his Ph.D. in Operations Research from DalTech (former TUNS), Dalhousie University, Halifax, Canada in 1991. He is currently an Associate Professor in Operations Research in the School of Engineering and Information Technology, University of New South Wales, UNSW@ADFA, Canberra, Australia. His main research areas are Applied Operations Research and Evolutionary Optimization. A/Prof. Sarker has published more than 170 refereed papers in journals, books and conference proceedings, and has edited 8 books on specialized topics. He is the lead author of the book '*Optimization Modeling: A Practical Approach*' published by Taylor and Francis in 2008, EMAIL: r.sarker@adfa.edu.au,.

Daryl Essam received his PhD in fractal image processing from the University of New South Wales at the Australian Defence Force Academy in 2000. His research focuses on new algorithms for genetic programming. In particular, the sub-fields of grammars, diversity, probabilistic approaches and multi-objective optimization. He is currently employed as a lecturer at the Australian Defense Force Academy, a campus of the University of New South Wales, Australia, EMAIL: d.essam@adfa.edu.au.